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ON A METHOD OF SOLVING PROBLEMS OF THE BENDING OF RODS AND PLATES OF PIECEWISE-CONSTANT STIFFNESS*

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By using an effective procedure /2/ expansions were constructed in /1/ of an arbitrary function in the eigenfunction integral of a boundary value problem for second-order differential equations given in an interval with different sections of stepwise-changing physical characteristics.

This procedure is applied below to construct the expansion of an arbitrary function in an eigenfunction series for fourth-order equations given in a finite interval with an arbitrary quantity of stepwise-varying characteristics. The finite integral transforms obtained in such a manner can be utilized in solving different bending problems for rods or plates with piecewise-varying stiffness, which enables us to formalize the procedure for solving specific problems. The need to solve differential equations given in a composite interval also arises in a study of the state of stress of layered elastic media. A general approach to the solution of problems of mechanics for layered media is proposed in /3/, and is based on extending the method of initial parameters and results in an effective calculational procedure.

1. We construct the expansion of the function $f(x)$ satisfying the Dirichlet conditions in a series of eigenfunctions $y(x)$ of the boundary value problem for a fourth order differential equation having stepwise varying physical constants in sections comprising the interval $[0, L]$. The boundary value problem in question can be written in the form

$$\begin{gathered}
y^{\mathrm{IV}}(x)+\lambda_1^4 y(x)=0 \quad (0<x<b_1), \quad \lambda_1^4=\alpha_1^4\lambda^4 \\
\cdots\cdots\cdots\cdots\cdots\cdots \\
y^{\mathrm{IV}}(x)+\lambda_j^4 y(x)=0 \quad (b_{j-1}<x<b_j), \quad \lambda_j^4=\alpha_j^4\lambda^4 \\
\cdots\cdots\cdots\cdots\cdots\cdots \\
y^{\mathrm{IV}}(x)+\lambda_k^4 y(x)=0 \quad (b_{k-1}<x<b_k), \quad \lambda_k^4=\alpha_k^4\lambda^4 \\
\left(b_p=\sum_{j=1}^{p} l_j,\ b_k=L\right)
\end{gathered} \tag{1.1}$$

Here λ are the eigenvalues of the problem under consideration, and the coefficients α_j describe the physical constants in a section j of length l_j.

The conditions on the section boundaries can be written in general form as follows:

$$\begin{gathered}
k_2^- y''(0)=k_1^- y'(0), \quad k_3^- y'''(0)=k_0^- y(0) \\
k_2^+ y''(L)=k_1^+ y'(L), \quad k_3^+ y'''(L)=k_0^+ y(L)
\end{gathered} \tag{1.2}$$

where the coefficients $k_i^{\pm}$ enable us to take into account different rod boundary conditions.

The conditions for the sections to be matched

$$y^{(i-1)}(b_j-0)=\mu_{ij}y^{(i-1)}(b_j+0) \quad (i=1,2,3,4) \tag{1.3}$$

must still be added to the conditions on the section boundaries ($y^{(i)}(x)$ is the derivative of order i of the function $y(x)$, and μ_{ij} are given coefficients).

To solve the problem according to /1, 2/, we consider the following partial differential equation

$$\frac{\partial^4 u(x,t)}{\partial x^4}=\alpha_j^4\frac{\partial u(x,t)}{\partial t} \quad (b_{j-1}<x<b_j) \tag{1.4}$$

with conditions on the section boundaries corresponding to (1.2) and (1.3) and the initial condition

$$u(0,x)=f(x) \tag{1.5}$$

instead of (1.1) with conditions (1.2) and (1.3).

Solving (1.4) by a Laplace transformation and taking account of condition (1.5), we obtain an ordinary differential equation with right-hand side for the Laplace transform $U(x, p)$

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$$U^{IV}(x, p) - p\,U(x, p) = -f(x) \quad (b_{j-1} < x < b_j) \tag{1.6}$$

The solution of (1.6) can be written in the form

$$U(x, p) = -\int_0^L G(x, \xi, p) f(\xi)\, d\xi \tag{1.7}$$

where $G(x, \xi, p)$ is Green's function of (1.6). The function $u(x, t)$ is determined by an inverse Laplace transform, and taking the initial condition (1.5) into account, we obtain the desired expansion of the function $f(x)$ (p_n are the roots of the denominator in Green's function)

$$f(x) = -\int_0^L \sum_{p_n} \operatorname{res} G(x, \xi, p_n) f(\xi)\, d\xi \tag{1.8}$$

Therefore, the construction of Green's function of (1.6) should be the next step in solving the problem. To do this, the solution of the homogeneous Eq. (1.6) must be obtained, which reduces to finding the eigenfunctions of the operator $U^{IV}(x, p)$ in this case. Starting with the right end of the interval, say, and moving sequentially from the last section to a certain section j, we find that

$$U_j(x, \lambda) = \sum_{i=1}^{2} E_{ik} \Phi_{ij}(x, \lambda) \tag{1.9}$$

$$\Phi_{1j}(x, \lambda) = \sum_{m=1}^{4} f_{2m-1, j}(\lambda) \mathbf{P}_m(\lambda_j(b_j - x)), \quad \lambda^4 = p \tag{1.10}$$

$$\Phi_{2j}(x, \lambda) = \sum_{m=1}^{4} f_{2m, j}(\lambda) \mathbf{P}_m(\lambda_j(b_j - x))$$

The Krylov functions S, T, U, V are denoted successively in terms of $\mathbf{P}_m$ in the expressions presented. The selection of the function $U_j(x, \nu)$ in the form (1.9) we can immediately determined the coefficient of $\mathbf{P}_m$ in (1.10) for the next section without solving the system of algebraic equations. Therefore, recursion relations have been obtained that connect the values of the coefficients $f_p(\lambda)$ successively, starting with the next section with number k

$$f_{1k} = k_3^+, \quad f_{2k} = k_2^+, \quad f_{ik} = 0 \quad (i = 3, 4, 5, 6), \quad f_{7k} = k_0^+, \quad f_{8k} = k_1^+$$

and later

$$f_{nj}(\lambda) = \delta_{m, j+1} \Phi_{n, j+1}(b_j) \quad (m = 1, 2, 3, 4) \tag{1.11}$$

$$\delta_{ij} = \left(\frac{\alpha_j}{\alpha_{j-1}}\right)^{i-1} \mu_{ij}, \quad \Phi_{i, j} = -\lambda_{j-1}^{-1} \Phi'_{i-2, j} \tag{1.12}$$

Similarly, taking account of the boundary conditions on the left-hand side and moving successively from the first section to section j, we obtain the eigenfunction

$$U_j(x, \lambda) = \sum_{i=1}^{2} C_{i1} \Psi_{ij}(x, \lambda) \tag{1.13}$$

$$\Psi_{1j}(x, \lambda) = \sum_{m=1}^{4} g_{2m-1, j}(\lambda) \mathbf{P}_m(\lambda_j(x - b_{j-1})) \tag{1.14}$$

$$\Psi_{2j}(x, \lambda) = \sum_{m=1}^{4} g_{2m, j}(\lambda) \mathbf{P}_m(\lambda_j(x - b_{j-1}))$$

For the first section

$$g_{11} = k_3^-, \quad g_{21} = k_2^-, \quad g_{i1} = 0 \quad (i = 3, 4, 5, 6)$$

and later

$$g_{n, j+1}(\lambda) = \delta_{m, j+1}^{-1} \Psi_{n, j}(b_j)$$

The functions $\Psi_{n,j}$ are determined from a formula similar to (1.12). Taking the conjugate of the functions $U_j(x, \lambda)$ defined by (1.9) and (1.13), we obtain Green's function of (1.6)

$$G_{jj} = \begin{cases} M_{jj}^1(\lambda, x, \xi) M_j^{-1}(\lambda) & (b_{j-1} < x < \xi < b_j) \\ M_{jj}^2(\lambda, \xi, x) M_j^{-1}(\lambda) & (b_{j-1} < \xi < x < b_j) \end{cases} \tag{1.15}$$

$$G_{ij}^1 = M_{ij}^1(\lambda, x, \xi) M_j^{-1}(\lambda)$$

$$(b_{i-1} < x < b_i < \ldots < b_{j-1} < \xi < b_j < \ldots < L)$$

$$G_{ji}^2 = M_{ji}^2(\lambda, \xi, x) M_i^{-1}(\lambda)$$

$$(b_{i-1} < \xi < b_i < \ldots < b_{j-1} < x < b_j < \ldots < L)$$

$$M^1_{ij} = Q_1{}^j \Psi_{1i}(x) - Q_2{}^j \Psi_{2i}(x), \quad M^2_{ji} = Q_3{}^i \Phi_{1j}(x) - Q_4{}^i \Phi_{2j}(x) \tag{1.16}$$

$$Q_1{}^j = \sum_{n=1}^{3} \Psi_{2n,j}(\xi) t_{n,j}(\xi), \quad Q_2{}^j = \sum_{n=1}^{3} \Psi_{2n-1,j}(\xi) t_{n,j}(\xi) \tag{1.17}$$

$$Q_3{}^j = \sum_{n=1}^{3} \Phi_{2n,j}(\xi) s_{n,j}(\xi), \quad Q_4{}^j = \sum_{n=1}^{3} \Phi_{2n-1,j}(\xi) s_{n,j}(\xi), \quad M_j = \lambda_j{}^3 W_j$$

$$W_j = \sum_{n=1}^{3} t_{nj}(\xi) s_{n+3,j}(\xi) + s_{nj}(\xi) t_{n+3,j}(\xi), \quad \omega_j(\xi) = \lambda_j{}^3 W_j' \tag{1.18}$$

Here

$$\begin{aligned} &t_{1j} = \Phi_{5j}\Phi_{4j} - \Phi_{3j}\Phi_{6j}, \quad t_{2j} = \Phi_{2j}\Phi_{5j} - \Phi_{1j}\Phi_{6j}, \\ &t_{3j} = \Phi_{2j}\Phi_{3j} - \Phi_{1j}\Phi_{4j} \\ &t_{4j} = \Phi_{1j}\Phi_{8j} - \Phi_{2j}\Phi_{7j}, \quad t_{5j} = \Phi_{3j}\Phi_{8j} - \Phi_{4j}\Phi_{7j}, \\ &t_{6j} = \Phi_{5j}\Phi_{8j} - \Phi_{6j}\Phi_{7j} \end{aligned} \tag{1.19}$$

The functions $s_{kj}(\xi)$ are obtained from $t_{kj}(\xi)$ by appropriate replacement of Φ by Ψ with the same subscripts.

Substituting (1.15) into (1.8), we write the desired expansion of the function $f(x)$

$$\begin{aligned} f(x) = -\sum_n \Big\{ &\omega_i^{-1}(\lambda_n) \sum_{p=1}^{j-1} \int_{b_{p-1}}^{b_p} M^{(2)}_{jp}(\lambda_n, \xi, x) f(\xi)\, d\xi + \\ &\omega_j^{-1}(\lambda_n) \Big[\int_{b_{j-1}}^{x} M^{(2)}_{jj}(\lambda_n, \xi, x) f(\xi)\, d\xi + \int_{x}^{b_j} M^{(1)}_{jj}(\lambda_n, x, \xi) f(\xi)\, d\xi + \\ &\sum_{p=j}^{k-1} \int_{b_p}^{b_{p+1}} M^{(1)}_{j,p+1}(\lambda_n, x, \xi) f(\xi)\, d\xi \Big\} \quad (b_{j-1} < x < b_j) \end{aligned} \tag{1.20}$$

2. Using the results obtained, we consider a rod with hinged supports that have two sections l_1 and l_2 of different physical characteristics α_1 and α_2. Setting $k = 2$, $k_1^- = k_1^+ = k_3^- = k_3^+ = 0$, and $k_0^- = k_0^+ = k_2^- = k_2^+ = 1$, we write the expression for Green's function for the case under consideration

$$G = \begin{cases} [T(u_1) r_1 - V(u_1) r_2]\, \omega_1^{-1} & (0 < x < \xi < l_1) \\ [\Phi_{11}(x) r_3 - \Phi_{21}(x) r_4]\, [2\omega_1^{-1}] & (0 < \xi < x < l_1) \\ R\, [T(u_1) r_5 - V(u_1) r_6]\, [2\omega_2^{-1}] & (0 < x < l_1 < \xi < L) \\ [T(\beta_3 - u_2) r_3 - V(\beta_3 - u_2) r_4]\, [2\omega_1^{-1}] & (0 < \xi < l_1 < x) \\ R\, [T(\beta_3 - u_2) r_7 - V(\beta_3 - u_2) r_8]\, \omega_2^{-1} & (0 < l_1 < \xi < x < L) \\ R\, [\Psi_{12}(x) r_5 - \Psi_{22}(x) r_6]\, [2\omega_2^{-1}] & (0 < l_1 < x < \xi < L) \end{cases} \tag{2.1}$$

Here

$$r_j = r_j(\xi) \ (j = 1, 2, \ldots, 8), \ \beta_i = \lambda_i l_i, \ u_i = \lambda_i x, \ v_i = \lambda_i \xi, \ R = \prod_{n=1}^{4} \delta_n$$

$$\omega_i = \lambda_i{}^3 W(\lambda) \ \ (i = 1, 2), \ \ \beta_3' = \lambda_2 L, \ \ L = l_1 + l_2, \ \ \delta_n = \mu_n \left(\frac{\alpha_2}{\alpha_1}\right)^{n-1}$$

The functions Φ_{i1} and Ψ_{i1} are defined by (1.10) and (1.14) with coefficients f and g that have the following form for the case under consideration

$$\begin{aligned} &f_{2m-1,1}(\lambda) = \delta_m \mathbf{P}_m(\beta_2), \quad f_{2m,2}(\lambda) = \delta_m Q_m(\beta_2) \quad (m = 1, 2, 3, 4) \\ &g_{2m-1,1}(\lambda) = \delta_m^{-1} \mathbf{P}_m(\beta_1), \quad g_{2m,2}(\lambda) = \delta_m^{-1} Q_m(\beta_1) \end{aligned} \tag{2.2}$$

Here $\mathbf{P}_m$ are the Krylov functions T, S, V, U and Q_m are V, U, T, S.

The remaining coefficients in (2.1) can also be expressed in terms of the Krylov functions and also $\Phi(\xi)$ and $\Psi(\xi)$:

$$\begin{aligned} &r_1 = V(v_1) t_{11} + U(v_1) t_{21} + T(v_1) t_{31}, \quad r_2 = T(v_1) t_{11} + \\ &\quad S(v_1) t_{21} + V(v_1) t_{31}, \\ &r_3 = c_2(v_1) \Phi_{61} + 2c_1(v_1) \Phi_{41} + c_4(v_1) \Phi_{21}, \quad r_4 = c_2(v_1) \Phi_{51} + \\ &\quad 2c_1(v_1) \Phi_{31} + c_4(v_1) \Phi_{11} \\ &r_5 = c_4(\beta_3 - v_2) \Psi_{22} + c_2(\beta_3 - v_2) \Psi_{62} + 2c_1(\beta_3 - v_2) \Psi_{42} \\ &r_6 = c_4(\beta_3 - v_2) \Psi_{12} + c_2(\beta_3 - v_2) \Psi_{52} + 2c_1(\beta_3 - v_2) \Psi_{32} \\ &r_7 = T(\beta_3 - v_2) s_{31} + U(\beta_3 - v_2) S_{21} + V(\beta_3 - v_2) s_{11}, \\ &r_8 = T(v_1) t_{11} + S(v_1) t_{21} + V(v_1) t_{31} \end{aligned}$$

The functions $t_{ij}(\xi)$ occurring here are defined by (1.19), while the functions c_i consist of combinations of hyperbolic functions

$$c_1(\beta) = \operatorname{sh}\beta \sin\beta_1,\ c_2(\beta) = \operatorname{ch}\beta \sin\beta - \operatorname{sh}\beta\cos\beta$$
$$c_4(\beta) = \operatorname{ch}\beta\sin\beta + \operatorname{sh}\beta\cos\beta,\ c_6(\beta) = \operatorname{ch}\beta\cos\beta$$

We also present an expression for the frequency function $W(\lambda)$, which, when equated to zero, enables us to determine the eigenvalues of the operator under consideration

$$W(\lambda) = 4^{-1}[\varepsilon_1 c_2(\beta_1) c_2(\beta_2) - \varepsilon_2 c_4(\beta_1) c_4(\beta_2) - 4\delta_1\delta_3 c_6(\beta_1) c_1(\beta_2) - 4\delta_2\delta_4 c_1(\beta_1) c_6(\beta_2)] \tag{2.3}$$

We now present the desired expansion of the function $f(x)$ for the interval consisting of the two sections

$$f(x) = \sum_{n=1}^{\infty} a_{1n}\sin\lambda_{1n}x - a_{2n}\operatorname{sh}\lambda_{1n}x \quad (0 \leqslant x \leqslant l_1) \tag{2.4}$$

$$f(x) = \sum_{n=1}^{\infty} b_{1n}\cdot\sin\lambda_{2n}(L-x) - b_{2n}\operatorname{sh}\lambda_{2n}(L-x) \quad (l_1 \leqslant x \leqslant L) \tag{2.5}$$

The series coefficients in these expressions are determined by formulas that are similar to the expressions for the coefficience of classical Fourier series, but more awkward

$$a_{1n} = m_1^{-1}\int_0^{l_1}(d_1q_1 - d_2^+q_2)f(\xi)\,d\xi - 4Rm_2^{-1}\int_{l_1}^{L}(d_3q_3 + d_4q_4)f(\xi)\,d\xi$$

$$a_{2n} = m_1^{-1}\int_0^{l_1}(d_5q_2 - d_2^-q_1)f(\xi)\,d\xi - 4Rm_2^{-1}\int_{l_1}^{L}(d_6q_3 + d_7q_4)f(\xi)\,d\xi$$

$$b_{1n} = 2m_1^{-1}\int_0^{l_1}(d_8q_1 + d_9q_2)f(\xi)\,d\xi + Rm_2^{-1}\int_{l_1}^{L}(d_{10}q_3 - d_{11}^+q_4)f(\xi)\,d\xi$$

$$b_{2n} = 2m_1^{-1}\int_0^{l_1}(d_{12}q_1 + d_{13}q_2)f(\xi)\,d\xi + Rm_2^{-1}\int_{l_1}^{L}(d_{14}q_4 - d_{11}^-q_3)f(\xi)\,d\xi$$

Here

$$q_1 = \sin\lambda_{1n}\xi,\ q_2 = \operatorname{sh}\lambda_{1n}\xi,\ q_3 = \sin\lambda_{2n}(L-\xi),\quad q_4 = \operatorname{sh}\lambda_{2n}(L-\xi)$$
$$m_1 = l_1B_1m_4 + m_3l_2\sqrt[4]{B_1^3B_2},\quad m_2 = l_2m_3B_2 + m_4l_1\sqrt[4]{B_1B_2^3}$$
$$m_3 = \varepsilon_1^+c_{122}c_{211} - \varepsilon_2^+c_{411}c_{622} - 2n_{13}c_{611}c_{422} + 2n_{24}c_{111}c_{222}$$
$$m_4 = \varepsilon_1^+c_{111}c_{222} - \varepsilon_2^+c_{422}c_{611} + 2n_{13}c_{211}c_{122} - 2n_{24}c_{411}c_{622}$$
$$d_1 = \varepsilon_1^+c_{222}c_{711}^+ + \varepsilon_2^-c_{422}c_{711}^- + 4n_{13}c_{122}c_{311} - 4n_{24}c_{622}c_{811}$$
$$d_2^\pm = \varepsilon_1^-c_{222} \pm \varepsilon_2^-c_{422},\quad d_3 = 2(\varepsilon_3^+c_{512} + \varepsilon_4^+c_{312}),\quad c_{1ik} = \operatorname{sh}\beta_i\sin\beta_k$$
$$d_4 = 2(\varepsilon_3^-c_{812} + \varepsilon_4^-c_{312}),\quad d_6 = -2(\varepsilon_3^-c_{321} + \varepsilon_4^-c_{821}),$$
$$c_{3ik} = \operatorname{ch}\beta_i\sin\beta_k$$
$$d_5 = \varepsilon_1^+c_{222}c_{711}^- - \varepsilon_2^+c_{422}c_{811} - 4n_{13}c_{122}c_{811} - 4n_{24}c_{622}c_{311}$$
$$d_7 = -2(\varepsilon_3^+c_{921} + \varepsilon_4^+c_{912}),\quad d_8 = \varepsilon_7^+c_{512} + \varepsilon_8^+c_{521},\quad c_{5ik} = \operatorname{sh}\beta_i\operatorname{ch}\beta_k$$
$$d_9 = \varepsilon_7^-c_{821} + \varepsilon_8^-c_{321},\quad d_{11}^\pm = \varepsilon_5^-c_{211} \pm \varepsilon_6^-c_{411},\quad c_{6ik} = \operatorname{ch}\beta_i\cos\beta_k$$
$$d_{10} = \varepsilon_5^+c_{211}c_{722}^+ + \varepsilon_6^-c_{411}c_{722}^- + 4n_{13}^{-1}c_{111}c_{322} - 4n_{24}^{-1}c_{611}c_{522}$$
$$d_{12} = \varepsilon_7^-c_{312} + \varepsilon_8^-c_{812},\quad d_{13} = \varepsilon_7^+c_{912} + \varepsilon_8^+c_{921},\quad c_{8ik} = \operatorname{sh}\beta_i\cos\beta_k$$
$$d_{14} = \varepsilon_5^+c_{211}c_{722}^- - \varepsilon_6^+c_{411}c_{722} - 4n_{24}^{-1}c_{611}c_{322} - 4n_{13}^{-1}c_{111}c_{822}$$
$$c_{9ik} = \cos\beta_i\sin\beta_k,\quad c_{2ik}^\pm = c_{3ik} \pm c_{8ik},\quad c_{4ik}^\pm = c_{1ik} \pm c_{6ik},\ n_{ik} = \delta_i\delta_k$$
$$\varepsilon_1^\pm = n_{12} \pm n_{34},\quad \varepsilon_2^\pm = n_{14} \pm n_{23},\quad \varepsilon_3^\pm = \delta_1^{-1} \pm \delta_3^{-1},\quad \varepsilon_4^\pm = \delta_2^{-1} \pm \delta_4^{-1}$$
$$\varepsilon_5^\pm = n_{12}^{-1} \pm n_{34}^{-1},\quad \varepsilon_6^\pm = n_{14}^{-1} \pm n_{23}^{-1},\quad \varepsilon_7^\pm = \delta_1 \pm \delta_3,\quad \varepsilon_8^\pm = \delta_2 \pm \delta_4$$

3. An an illustration of the application of the formulas obtained we consider the forced vibrations of a beam with two sections of length l_1 and l_2, which have different respective stiffnesses B_1 and B_2. As is well-known, the modes of steady forced vibrations of a beam satisfy the following equation in this case:

$$\begin{aligned} \varphi^{IV}(x) - \lambda_1^4\varphi(x) &= q(x) \quad (0 \leqslant x \leqslant l_1) \\ \varphi^{IV}(x) - \lambda_2^4\varphi(x) &= q(x) \quad (l_1 \leqslant x \leqslant L) \end{aligned} \tag{3.1}$$

Here

$$\lambda_i^4 = p^2\left(\frac{m}{B}\right)_i \tag{3.2}$$

(p is the frequency of natural vibrations of the beam, and m_i is the mass per unit length of the sections).

To solve the equation obtained we expand the functions $\varphi(x)$ and $q(x)$ in the series (2.4) and (2.5) and, as usual, substitute them into (3.1) to find that the coefficients of the series (2.4) and (2.5) for the function $\varphi(x)$ are expressed in terms of the known coefficients c_{in} and d_{in} according to the relationships

$$a_{in} = \frac{c_{in}}{\lambda_{1n}^4 + a_1^4}, \quad b_{in} = \frac{d_{in}}{\lambda_{1n}^4 + a_1^4} \quad (i = 1, 2) \tag{3.3}$$

$$a_{in} = \frac{c_{in}}{\lambda_{2n}^4 + a_2^4}, \quad b_{in} = \frac{d_{in}}{\lambda_{2n}^4 + a_2^4} \quad (i = 3, 4)$$

The coefficients c_{in} and d_{in} are determined from (2.6) for $f(\xi) = q(\xi)$.

To evaluate the coefficients by means of (3.3), it is also necessary to know the eigenvalues λ which depend on the relationships between the characteristics α_1 and α_2. We assume the stiffness of one section to be double the stiffness of the other $(B_2 : B_1 = 2)$ for identical constant mass, and each section to equal half the span of the beam. The eigenvalues λ_i for this case, as obtained in the solution of (3.2), are presented below

i	1	2	3	4	5	6	7
λ_{1i}	2,64	5,28	7,92	10,56	13,2	15,85	18,49
λ_i	2,83	5,79	8,54	11,55	14,27	17,29	20,02
λ_{2i}	3,14	6,28	9,42	14,27	15,70	18,85	21,99

We give here the values of λ_{ij} from beams with stiffnesses B_1 and B_2, respectively. Halving the stiffness of one of the sections results in a change of approximately 15—25% in the rod vibration frequency as compared with a rod of constant stiffness. The frequencies of natural vibrations for a rod with the above-mentioned relationship of variable stiffness can be determined from (3.2) by using the middle row.

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